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283. Proposed by B. F. FINKEL, Ph. D., Drury College.

By means of the calculus, determine the angle of minimum deviation of a ray of monochromatic light in passing through a triangular prism.

Solution by C. N. SCHMALL, New York City.

We have, with the usual notation,

$$\sin\phi=\mu\sin\phi' \ldots (1),$$
$$\sin\psi=\mu\sin\psi' \ldots (2),$$
$$\phi'+\psi'=\text{a constant, } k, \text{ say} \ldots (3),$$
$$u=\phi+\psi=\text{a minimum} \ldots (4).$$

Solving (1) and (2), we have, $\phi'=\sin^{-1}(\sin\phi/\mu)$, $\psi'=\sin^{-1}(\sin\psi/\mu)$. Substituting in (3), we have

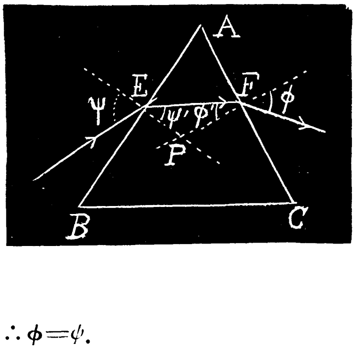
$$\sin^{-1}\left(\frac{\sin\phi}{\mu}\right)+\sin^{-1}\left(\frac{\sin\psi}{\mu}\right)=k \ldots (5).$$

Differentiating with respect to ϕ and reducing slightly, we have

$$\frac{\cos\phi}{\sqrt{(\mu^2-\sin^2\phi)}}+\frac{\cos\psi}{\sqrt{(\mu^2-\sin^2\psi)}}\frac{d\phi}{d\psi}=0 \ldots (6).$$

Differentiating (4), we have

$$\frac{du}{d\phi}=1+\frac{d\psi}{d\phi}=0 \ldots (7).$$



Equating the values of $\frac{d\psi}{d\phi}$, found from (6) and (7), we have finally,

$$\frac{\cos^2\phi}{\mu^2-\sin^2\phi}=\frac{\cos^2\psi}{\mu^2-\sin^2\psi}. \quad \therefore \phi=\psi.$$

Now, the deviation is $D=(\psi-\psi')+(\phi-\phi')=2(\phi-\phi')$, (since $\psi=\phi$)...(8).

But this will not be the case unless the $\triangle ABC$ is isosceles. Then, if θ be the angle of the prism, that is, angle A, we have

$$\phi'=\tfrac{1}{2}\theta, \text{ and from (1), } \mu=\frac{\sin\phi}{\sin\phi'}\ldots(9),$$

but from (8), $\phi=\frac{1}{2}(D+2\phi')=\frac{1}{2}(D+\theta)$. Hence, substituting in (9), we have,

$$\mu=\frac{\sin\frac{1}{2}(D+\theta)}{\sin\frac{1}{2}\theta}\ldots(10),$$

which gives the relation between the minimum deviation D, the angle of the prism θ, and the index of refraction μ.

Also solved by G. B. M. Zerr and J. Scheffer.

284. Proposed by L. H. McDONALD, M. A., Ph. D., Sometimes Tutor at Cambridge, Jersey City, N. J.

Inscribe the triangle of maximum area in a given circle.

Solution by PROFESSOR F. L. GRIFFIN, Williams College.

Let the diameter, of length d, through any vertex make angles α and β with the two sides of the triangle meeting there. Then two sides of the triangle are $d\cos\alpha$ and $d\cos\beta$; and their included angle is $\alpha+\beta$. [$\alpha-\beta$ is excluded, since a triangle contained in a semi-circle clearly can not be the maximum.] Now the area is $\frac{1}{2}(d\cos\alpha).(d\cos\beta).\sin(\alpha+\beta)$; so that we have to render a maximum the function $F(\alpha,\beta)\equiv\cos\alpha.\cos\beta.\sin(\alpha+\beta)$.

$$\text{But } \frac{\partial F}{\partial\alpha}=\cos\beta\,(\cos\alpha.\cos\overline{\alpha+\beta}-\sin\alpha.\sin\overline{\alpha+\beta})=\cos\beta.\cos(2\alpha+\beta),$$

$$\frac{\partial F}{\partial\beta}=\cos\alpha.\cos(2\beta+\alpha).$$

Equating these to zero, we obtain since $\cos\alpha\neq0$, $\cos\beta\neq0$ if there is actually a triangle:

$$2\alpha+\beta=\tfrac{1}{2}\pi,\ 2\beta+\alpha=\tfrac{1}{2}\pi, \text{ whence } \alpha=\beta=\tfrac{1}{6}\pi.$$

$$\text{Also, } \frac{\partial^2F}{\partial\alpha^2}=-2\sin(2\alpha+\beta)\cos\beta,\quad \frac{\partial^2F}{\partial\alpha\,\partial\beta}=-\sin\overline{2\alpha+2\beta},$$

$$\frac{\partial^2F}{\partial\beta^2}=-2\sin(2\beta+\alpha).\cos\alpha;\text{ whence, } \left[\frac{\partial^2F}{\partial\alpha^2}.\frac{\partial^2F}{\partial\beta^2}-\left(\frac{\partial^2F}{\partial\alpha\,\partial\beta}\right)^2\right]_{\alpha=\beta=\frac{1}{6}\pi}=\tfrac{3}{2},$$

the positive result together with the negative value of $\partial^2F/\partial\alpha^2$ showing a maximum value of $F(\alpha,\beta)$.